Abstract

In many countries, inflation has become less responsive to domestic factors and more responsive to global factors over the past decades. We study the linkages between domestic inflation and global liquidity (money and household balances) and argue that it is important for inflation modeling and forecasting. We introduce money and credit markets into the workhorse open-economy New Keynesian model. With this framework, we show that: (i) an efficient forecast of domestic inflation must be based solely on domestic and foreign slack, and (ii) global liquidity (either global money or global credit) is tied to global slack in equilibrium. Then, motivated by the theory, we empirically evaluate the performance of open-economy Phillips curve-based forecasts constructed using global liquidity measures (such as G7 credit growth and G7 money supply growth) instead of global slack as predictive regressors. Using 50 years of quarterly data, we document that these global liquidity variables perform significantly better than the domestic variable counterparts and outperform in practice the poorly-measured indicators of global slack that global liquidity proxies for.

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Online Appendix

A The Building Blocks of the Model

The framework can be considered an extension to the (closed-economy) money-in-the-utility New Keynesian model in Galí (2008) where households can gain utility from real credit as well as cash balances to generate the demand side of the money and credit market. In turn, the supply side of the money and credit markets is based on the balanced sheets of the monetary authority and the banking system. With a role for credit, our paper contributes to the theoretical understanding of the linkages between inflation, global economic activity and global liquidity which has not been studied in earlier open-economy New Keynesian models such as Kabukcuoglu and Martínez-García (2018), as well as other related open-economy New Keynesian models such as the model of Clarida et al. (2002), among others. Here we describe the main features of the open-economy New Keynesian framework with a credit channel maintaining the symmetry in the structure of both countries between households, firms, the banking system, and the central banks. We illustrate the model with the first principles from the Home country unless otherwise noted, and use the superscript * to denote Foreign country variables (or parameters).

A.1 Households’ Optimization

The lifetime utility of the representative household in the Home country is additively separable in consumption, $C_t$, a real liquidity bundle of cash and credit, $X_t$, and labor, $N_t$, i.e.,

$$
\sum_{\tau=0}^{+\infty} \beta^\tau \mathbb{E}_t \left[ \frac{1}{1-\gamma} (C_{t+\tau})^{1-\gamma} + \frac{\chi}{1-\zeta} (X_{t+\tau})^{1-\zeta} - \frac{\kappa}{1+\varphi} (N_{t+\tau})^{1+\varphi} \right],
$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution on consumption, $\zeta > 0$ determines the inverse elasticity of the liquidity bundle, and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. The scaling factors $\chi > 0$ and $\kappa > 0$ pin down liquidity and labor in steady state.

We adapt the money-in-the-utility-function approach to recognize that transactions require households to retain a liquidity position and to incorporate the gains associated with liquidity into the model. We also recognize that real money balances (or real cash) is not the only way that liquidity gains can be achieved, access to real credit is another way. Hence, we assume that the liquidity bundle, $X_t$, is a non-separable constant-elasticity of substitution (Armington) aggregator between real money balances (real cash), $\frac{Z_t}{P_t}$, and real credit, $\frac{L_t}{P_t}$, given by,

$$
X_t = \left( \mu \right)^{\frac{v}{\nu}} \left( \frac{Z_t}{P_t} \right)^{\frac{v-1}{\nu}} + (1-\mu)^{\frac{v}{\nu}} \left( \frac{L_t}{P_t} \right)^{\frac{v-1}{\nu}},
$$

where $\nu > 0$ denotes the elasticity of substitution between real money balances (cash) and real credit and $0 < \mu \leq 1$ captures the relative weight of real money balances and real credit in the household’s per-period utility gains from liquidity. Only in the special case in which real balances and real credit are perfect substitutes (when $\nu \to \infty$), simple aggregation of both suffices to measure liquidity.\(^1\) In the special case

\(^1\)Whenever $v$ approaches infinity, real balances and real credit become perfect substitutes; whenever $v$ approaches zero, they are
where $\mu = 1$, the liquidity position reduces to real balances which is the standard assumption in the existing money-in-the-utility-function literature.

The representative household maximizes its lifetime utility in (1) subject to the following sequence of budget constraints which holds across all states of nature $\omega_t \in \Omega$, i.e.,

$$ P_t C_t + \int_{\omega_{t+1} \in \Omega} Q_t (\omega_{t+1}) B_t^H (\omega_{t+1}) + S_t \int_{\omega_{t+1} \in \Omega} Q_t^* (\omega_{t+1}) B_t^F (\omega_{t+1}) + Z_t + D_t - L_t $noveq{3}

$$ \leq W_t N_t + P_t T_t + B_{t-1}^H (\omega_t) + S_t B_{t-1}^F (\omega_t) + Z_{t-1} - (1 + i_{t-1}) D_{t-1} - (1 + i_{L,t-1}) L_{t-1}, $$

where $W_t$ is the nominal wage in the Home country, $P_t$ is the Home consumer price index (CPI), $T_t$ is a nominal lump-sum tax (or transfer) imposed by the Home government, and $P_t R_t$ are (per-period) nominal profits from all firms producing the Home varieties and the Home banking system. We denote the fully-flexible bilateral nominal exchange rate as $S_t$ indicating the units of the currency of the Home country that can be obtained per each unit of the Foreign country currency at time $t$.

The representative household’s budget constraint includes a portfolio of one-period Arrow-Debreu securities (contingent bonds) traded internationally, issued by the governments of both countries each in their own currencies and in zero-net supply. That is, the pair $\{ B_t^H (\omega_{t+1}), B_t^F (\omega_{t+1}) \}$ refers to the portfolio of contingent bonds issued by both countries held by the representative household of the Home country. Access to a full set of internationally-traded, one-period Arrow-Debreu securities completes the local and international asset markets recursively. The prices of the Home and Foreign contingent bonds expressed in their currencies of denomination are denoted $Q_t (\omega_{t+1})$ and $Q_t^* (\omega_{t+1})$, respectively.\(^2\)

The budget constraint also takes into account that, to achieve his optimal liquidity position, the representative household holds non-interest-bearing cash or nominal money balances, $Z_t$, but it also makes nominal deposits with the banking system, $D_t$, that earn a guaranteed nominal return of $i_t$ while at the same time taking loans from the banking system, $L_t$, at a net interest rate of $i_{L,t}$. The function of the banking system that we highlight here is that of a liquidity provider that transforms household’s savings into liquidity in order to facilitate the functioning of the payment system. Furthermore, we also assume that liquidity is locally-provided—cash issued by the domestic central bank only circulates within each country’s borders and domestic loans are supplied solely by the locally-based banking system (abstracting from issues like cross-border loans, global currencies). Similarly, we define the problem of each household in the Foreign country.

**Households’ asset demand equations.** Under complete asset markets, standard no-arbitrage results imply that $Q_t (\omega_{t+1}) = \frac{S_t}{S_{t+1} (\omega_{t+1})} Q_t^* (\omega_{t+1})$ for every state of nature $\omega_t \in \Omega$. Hence, Home and Foreign households can efficiently share risks domestically as well as internationally—this implies that the intertemporal marginal rate of substitution is equalized across countries at each possible state of nature, and accordingly it follows that:

$$ \beta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \frac{P_{t-1}}{P_t} = \beta \left( \frac{C_t^*}{C_{t-1}^*} \right)^{-\gamma} \frac{P_{t-1}^* S_{t-1}}{P_t^* S_t}. $$noveq{4}

\(^2\)The price of each bond in the currency of the country who did not issue the bond is converted at the prevailing bilateral exchange rate with full exchange rate pass-through under the law of one price (LOOP).
We define the bilateral real exchange rate as \( RS_t \equiv \frac{S_t P_t}{T_t} \), so by backward recursion the perfect international risk-sharing condition in (4) implies that,

\[
RS_t = v \left( \frac{C_t}{C_t} \right)^{-\gamma},
\]

where \( v \equiv \frac{S_0 P_0}{S_0 T_0} \left( \frac{C_0}{C_0} \right)^\gamma \) is a constant that depends on initial conditions. If the initial conditions correspond to those of the symmetric steady state, then the constant \( v \) is equal to one.

Home country household’s savings on a one-period, non-contingent nominal deposit in the Home country banking system result in the following standard stochastic Euler equation:

\[
\frac{1}{1 + i_t} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right],
\]

where \( i_t \) is the risk-free Home nominal interest rate (or, simply put, the nominal return on deposits in the Home banking system). This is equivalent to the yield on a redundant one-period, non-contingent nominal bond in the Home country which can be synthetically computed from the price of the contingent Arrow-Debreu securities in the Home country.

From the household’s first-order conditions on nominal balances of cash and credit (\( Z_t \) and \( L_t \)), we obtain the following pair of equilibrium conditions that dictate the demand for cash and credit in the Home country,

\[
\chi(\mu)^{\frac{1}{\nu}} \left( \frac{Z_t}{P_t} \right)^{-\frac{1}{\nu}} = (X_t)^{\frac{\nu-1}{\nu}} \frac{C_t}{C_t}^{-\gamma} \frac{i_t}{1 + i_t},
\]

\[
\chi(1 - \mu)^{\frac{1}{\nu}} \left( \frac{L_t}{P_t} \right)^{-\frac{1}{\nu}} = (X_t)^{\frac{\nu-1}{\nu}} \frac{C_t}{C_t}^{-\gamma} \left( \frac{i_{L,t} - i_t}{1 + i_t} \right).
\]

Taking the ratio of both equilibrium conditions it follows that,

\[
\frac{L_t}{P_t} = \frac{\mu}{1 - \mu} \left( \frac{i_t}{i_{L,t} - i_t} \right)^{\nu} \frac{Z_t}{P_t},
\]

which shows that—in an interior solution where both cash and credit are used—the demand of real credit must be equal to a multiplier over the demand for real money balances. The multiplier in (9) depends on the risk-free Home nominal interest rate, \( i_t \), and on the spread between the loan rate and the rate paid on deposits, \( i_{L,t} - i_t \).

Replacing (9) into (2), we can express the liquidity position of the households in the Home country as proportional to his holdings of real balances, i.e.,

\[
X_t = \left[ (\mu)^{\frac{1}{\nu}} + (1 - \mu)^{\frac{1}{\nu}} \left( \frac{\mu}{1 - \mu} \right)^{\nu - 1} \left( \frac{i_t}{i_{L,t} - i_t} \right)^{\nu - 1} \right]^{\frac{1}{\nu}} \left( \frac{Z_t}{P_t} \right).
\]

Combining this expression for the real liquidity bundle with the first-order condition on real balances in
(7), we obtain that:

\[
\chi(\mu)^{\frac{1}{2}} \left( \frac{Z_t}{P_t} \right)^{-\xi} = (C_t)^{-\gamma} \left( (\mu)^{\frac{1}{2}} + (1 - \mu)^{\frac{1}{2}} \left( \frac{\mu}{1 - \mu} \right)^{\frac{v-1}{2}} \left( \frac{i_t}{i_{L,t} - i_t} \right)^{\frac{v-1}{2}} \right)^{\frac{\xi}{\gamma}} \left( \frac{i_t}{1 + i_t} \right),
\]

which defines the demand for real money balances in the model. Whenever \( \xi = \gamma \), the expression for money demand in (11) can be seen as a special case of the quantity theory of money equation where consumption expenditures \( (P_tC_t) \) are related to money holdings (cash holdings) with a scaling factor—akin to the velocity of money in the quantity theory of money equation—that generally depends on both the risk-free Home nominal interest rate, \( i_t \), and the spread between the loan rate and the risk-free rate, \( i_{L,t} - i_t \).

Equations (9) and (11) fully describe the demand-side of the credit and money markets.

**Household’s labor supply and consumption demand equations.** We assume within-country labor mobility—although labor remains immobile across countries—ensuring that wages equalize across firms in a given country but not necessarily across countries. From the household’s first-order conditions we obtain a labor supply equation of the following form,

\[
\frac{W_t}{P_t} = \kappa (C_t)^{\gamma} (N_t)^{\phi}.
\]

With flexible wages, all households are paid the same nominal wage rate, \( W_t \), and work the same hours, \( N_t \), in equilibrium.

The consumption of the representative household in the Home country, \( C_t \), is given by a nested CES aggregator of both countries’ bundle of varieties. The consumption CES index for the Home representative household is defined as:

\[
C_t = \left[ (1 - \xi)^{\frac{1}{\sigma}} \left( C_t^H \right)^{\frac{\sigma - 1}{\sigma}} + (\xi)^{\frac{1}{\sigma}} \left( C_t^F \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}},
\]

where \( \sigma > 0 \) is the elasticity of substitution between the consumption bundle of Home-produced goods consumed in the Home country, \( C_t^H \), and the Home consumption bundle of the Foreign-produced goods, \( C_t^F \). Similarly, the CES aggregator for the Foreign country is defined as:

\[
C_t^F = \left[ (\xi)^{\frac{1}{\sigma}} \left( C_t^H \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \xi)^{\frac{1}{\sigma}} \left( C_t^F \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}},
\]

where \( C_t^H \) and \( C_t^F \) are respectively the consumption bundle of Foreign-produced goods and of Home-produced goods for the Foreign country household. The share of imported goods in the consumption basket of each country is given by \( \xi \) and satisfies that \( 0 \leq \xi \leq \frac{1}{2} \), allowing for local-consumption bias whenever \( \xi < \frac{1}{2} \) (since each country produces an equal share of varieties). The consumption CES sub-indexes aggregate the consumption of the representative household over the bundle of differentiated varieties produced
by each country and are defined as follows:

\[
C^H_i = \left[ \int_0^1 C_i (h)^{\frac{1}{1-\theta}} dh \right]^{\frac{\theta}{1-\theta}}, \quad C^*_i = \left[ \int_0^1 C^*_i (f)^{\frac{1}{1-\theta}} df \right]^{\frac{\theta}{1-\theta}},
\]

(15) \hspace{1cm}

\[
C^{H*}_i = \left[ \int_0^1 C^{H*}_i (h)^{\frac{1}{1-\theta}} dh \right]^{\frac{\theta}{1-\theta}}, \quad C^{F*}_i = \left[ \int_0^1 C^{F*}_i (f)^{\frac{1}{1-\theta}} df \right]^{\frac{\theta}{1-\theta}},
\]

(16)

where \( \theta > 1 \) is the elasticity of substitution across the differentiated varieties within a country.

The consumption price indexes (CPIs) that correspond to this specification of consumption preferences are,

\[
P_i = \left[ (1 - \xi) \left( P^H_i \right)^{1-\sigma} + \xi \left( P^F_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad P^*_i = \left[ \xi \left( P^{H*}_i \right)^{1-\sigma} + (1 - \xi) \left( P^{F*}_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}},
\]

and,

\[
P^H_i = \left[ \int_0^1 P_i (h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}, \quad P^F_i = \left[ \int_0^1 P_i (f)^{1-\theta} df \right]^{\frac{1}{1-\theta}},
\]

(18) \hspace{1cm}

\[
P^{H*}_i = \left[ \int_0^1 P^*_i (h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}, \quad P^{F*}_i = \left[ \int_0^1 P^*_i (f)^{1-\theta} df \right]^{\frac{1}{1-\theta}},
\]

(19)

where \( P^H_i \) and \( P^{H*}_i \) are the price sub-indexes for the bundle of locally-produced varieties in the Home and Foreign countries, respectively. The price sub-index \( P^F_i \) represents the Home country price of the bundle of Foreign varieties, while \( P^{F*}_i \) is the Foreign country price for the bundle of Home varieties. The price of variety \( h \) produced in the Home country is expressed as \( P_i (h) \) and \( P^*_i (h) \) in units of the Home and Foreign currency, respectively. Similarly, the price of variety \( f \) produced in the Foreign country is quoted in both countries as \( P_i (f) \) and \( P^*_i (f) \), respectively.

Each household decides how much to allocate to the different varieties of goods produced in each country. Given the structure of preferences given here, the utility maximization problem implies that the household’s demand for each variety is given by:

\[
C_i (h) = \left( \frac{P_i (h)}{P^H_i} \right)^{-\theta} C^H_i, \quad C_i (f) = \left( \frac{P_i (f)}{P^F_i} \right)^{-\theta} C^F_i,
\]

(20) \hspace{1cm}

\[
C^*_i (h) = \left( \frac{P^*_i (h)}{P^{H*}_i} \right)^{-\theta} C^{H*}_i, \quad C^*_i (f) = \left( \frac{P^*_i (f)}{P^{F*}_i} \right)^{-\theta} C^{F*}_i,
\]

(21)

while the demand for the bundle of varieties produced by each country is simply equal to,

\[
C^H_i = (1 - \xi) \left( \frac{P^H_i}{P_i} \right)^{-\sigma} C_i, \quad C^F_i = \xi \left( \frac{P^F_i}{P_i} \right)^{-\sigma} C_i,
\]

(22) \hspace{1cm}

\[
C^{H*}_i = \xi \left( \frac{P^{H*}_i}{P^*_i} \right)^{-\sigma} C^*_i, \quad C^{F*}_i = (1 - \xi) \left( \frac{P^{F*}_i}{P^*_i} \right)^{-\sigma} C^*_i.
\]

(23)

These equations relate the demand for each variety—whether produced domestically or imported—to the aggregate consumption of the country.

The optimization problem of the representative household of the Home country also satisfies the budget
profits, i.e., \( p_i < p_t \). In each period, every firm receives either a signal to maintain their prices with probability 0 \( \alpha = \{0, 1\} \), or a signal to re-optimize them with probability 1 \( \beta = \{0, 1\} \). Hence, it follows naturally that the conforming price sub-indexes in both countries computed for the same bundle of varieties must satisfy that \( P_{t} = P_{H_t}^{f} = P_{F_t}^{f} \).

The bilateral terms of trade \( T = \frac{p_t}{p_{H_t}} \) define the Home country value of the imported bundle of goods from the Foreign country in Home currency units relative to the Foreign value of the bundle of the Home country’s exports (quoted in the currency of the Home country at the prevailing bilateral nominal exchange rate). Under the LOOP, terms of trade can be expressed as,

\[
T = \frac{p_t}{p_{H_t}} = \frac{p_t^{H_t}}{p_{H_t}^{H_t}} = \frac{p_t^{F_t}}{p_{H_t}^{F_t}}.
\]

Even though the LOOP holds, the assumption of local-product bias in consumption introduces deviations from purchasing power parity (PPP) at the level of the consumption basket. For this reason, \( P_t \neq S_t P_t^* \) and, therefore, the bilateral real exchange rate between both countries deviates from one—i.e., \( R_{S_t} = \frac{S_t^*}{P_t} = \left(\frac{1 - \phi}{1 - \phi}\right)^{1/\sigma} \neq 1 \) if \( \phi \neq 1/2 \).

Given households’ preferences in each country, the demand for any variety \( h \in [0, 1] \) produced in the Home country is given as,

\[
Y_t(h) = C_t(h) + C_t^*(h) = (1 - \phi) \left( \frac{P_t}{P_t^*} \right)^{-\phi} \left( \frac{P_t^H}{P_t^F} \right)^{-\phi} C_t + \phi \left( \frac{P_t}{P_t^*} \right)^{-\phi} \left( \frac{P_t^H}{P_t^F} \right)^{-\phi} C_t^*.
\]

Similarly, we derive the demand for each variety \( f \in [0, 1] \) produced by the Foreign firms. Firms maximize profits subject to a partial adjustment rule à la Calvo (1983) at the variety level (that is, subject to sticky prices). In each period, every firm receives either a signal to maintain their prices with probability \( 0 < \alpha < 1 \) or a signal to re-optimize them with probability \( 1 - \alpha \). At time \( t \), the re-optimizing firm producing variety \( h \) in the Home country chooses a price \( \tilde{p}_t(h) \) optimally to maximize the expected discounted value of its profits, i.e.,

\[
\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ (\alpha \beta)^{T} \left( \frac{C_{t+\tau}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+\tau}} \left[ Y_{t+\tau}(h) \left( \tilde{p}_t(h) - (1 - \phi) MC_{t+\tau} \right) \right] \right\},
\]

subject to the constraint that the aggregate demand given in (25) is always satisfied at the set price \( \bar{P}_t(h) \) as
long as it remains in place (even when this implies per-period losses for the firm). \( \tilde{Y}_{t, t+\tau} (h) \) indicates the demand for consumption of the variety \( h \) produced in the Home country at time \( t + \tau \) \((\tau > 0) \) whenever the prevailing prices remain unchanged since time \( t \)—i.e., whenever \( P_{t+s} (h) = \tilde{P}_t (h) \) for all \( 0 \leq s \leq \tau \). An analogous problem describes the optimal price-setting behavior of the re-optimizing firms in the Foreign country.

Hence, the (before-subsidy) nominal marginal cost in the Home country \( MC_t \) can be expressed as,

\[
MC_t = \left( \frac{W_t}{A_t} \right),
\]

(27)

where the Home productivity (TFP) shock is denoted by \( A_t \). A similar expression holds for the Foreign country’s (before-subsidy) nominal marginal cost. Productivity shocks are described with the following bivariate stochastic process,

\[
A_t = (A)^{1-\delta} (A_{t-1})^{\delta} \left( A_{t-1}^{*} \right)^{\delta_{a,a^*}} \epsilon_t^{a},
\]

(28)

\[
A_t^{*} = (A)^{1-\delta} (A_{t-1})^{\delta} \left( A_{t-1}^{*} \right)^{\delta_{a,a^*}} \epsilon_t^{a^*},
\]

(29)

\[
\begin{pmatrix} \epsilon_t^{a} \\ \epsilon_t^{a^*} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^{2} & \rho_{a,a^*}\sigma_a^{2} \\ \rho_{a,a^*}\sigma_a^{2} & \sigma_{a^*}^{2} \end{pmatrix} \right),
\]

(30)

where \( A \) is the unconditional mean of the process (normalized to one), \( \delta_a \) and \( \delta_{a,a^*} \) capture the persistence and cross-country spillovers of the bivariate process assumed to be stationary, and \( (\epsilon_t^{a}, \epsilon_t^{a^*})^T \) is a vector of Gaussian innovations with a common variance \( \sigma_a^{2} > 0 \) and possibly correlated across both countries \( 0 < \rho_{a,a^*} < 1 \).

The optimal pricing rule of the re-optimizing firm \( h \) of the Home country at time \( t \) is given by,

\[
\tilde{P}_t (h) = \left( \frac{\theta}{\theta + 1} (1 - \phi) \right) \sum_{T=0}^{+\infty} (\alpha \beta)^T \mathbb{E}_t \left[ \left( \begin{pmatrix} \left( C_{t+\tau} \right)^{-1} \\ P_{t+\tau} \end{pmatrix} \right) \tilde{Y}_{t+\tau} (h) MC_{t+\tau} \right],
\]

(31)

where \( \phi \) is a time-invariant labor subsidy which is proportional to the nominal marginal cost \( MC_{t+\tau} \). An analogous expression can be derived for the optimal pricing rule of the re-optimizing firm \( f \) in the Foreign country to pin down \( \tilde{P}_t (f) \).

Given the inherent symmetry of the Calvo-type pricing scheme, the price sub-indexes in both countries for the bundles of varieties produced locally, \( P_t^{H} \) and \( P_t^{F} \), respectively, evolve according to the following pair of laws of motion,

\[
\begin{align*}
\left( P_t^{H} \right)^{1-\theta} & = \alpha \left( P_{t-1}^{H} \right)^{1-\theta} + (1 - \alpha) \left( \tilde{P}_t (h) \right)^{1-\theta}, \\
\left( P_t^{F} \right)^{1-\theta} & = \alpha \left( P_{t-1}^{F} \right)^{1-\theta} + (1 - \alpha) \left( \tilde{P}_t (f) \right)^{1-\theta},
\end{align*}
\]

(32)

(33)

linking the current-period price sub-index to the previous-period price sub-index and to the symmetric pricing decision taken by all the re-optimizing firms during the current period. Then, the LOOP relates these price sub-indexes to \( P_t^{H} \) and \( P_t^{F} \) with full pass-through of the bilateral nominal exchange rate \( S_t \).

In order to characterize the allocation in the counterfactual case where nominal rigidities are removed
and prices are fully flexible, we must replace the optimal pricing rule in (31) with the standard rule under perfect competition and flexible prices, i.e.,

\[ \tilde{P}_t(h) = MC_t, \]  

(34)

for each firm \( h \) in the Home country at time \( t \). Solving the model under this alternative price-setting rule defines the equilibrium allocation—and, in particular, the output and real interest rates—that would prevail in the frictionless environment subject to otherwise the same shocks. We refer to output and real interest rates in this frictionless counterfactual case as the economy’s output potential and natural (real interest) rate, respectively.

### A.3 The Banking System and the Policy Framework

As noted before we introduce a simplified banking system in the model whose unique function is to transform local household’s savings into household liquidity via credit. We abstract from other relevant functions of the banking system such as maturity transformation and risk sharing in this environment, but we view those as important areas for our future research. We further assume that the banking system is perfectly competitive and describe with a representative bank in each country solely owned by the local household.

The banking system acts as a financial lever for monetary policy. Hence, we need to be more explicit about the policy framework and implementation whenever both money and credit markets are taken into account. We start describing a rather general—but stylized—description of the balance sheet of the central bank and the banking system in the Home country to illustrate their linkages:

<table>
<thead>
<tr>
<th>Central Bank</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>( B^m_t ) (Holdings of government bonds)</td>
<td>( Z_t ) (Currency in circulation)</td>
</tr>
<tr>
<td>( F_t ) (Loans to commercial banks)</td>
<td>( U_t ) (Required and excess reserves)</td>
</tr>
<tr>
<td></td>
<td>( = Z_t + U_t = MB_t ) (Monetary Base)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Banking System</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>( U_t ) (Required and excess reserves)</td>
<td>( D_t ) (Deposits)</td>
</tr>
<tr>
<td>( \int_{\omega_{t+1} \in \Omega} Q_t (\omega_{t+1}) B^b_t (\omega_{t+1}) ) (Holdings of government bonds)</td>
<td>( F_t ) (central bank loans)</td>
</tr>
<tr>
<td>( L_t ) (Loans to households)</td>
<td></td>
</tr>
</tbody>
</table>

Notice that an important simplification on these balance sheets is that we abstract from including the central bank’s and the banking system’s equity on the liability side. This is because we assume that the central bank has the full backing of the fiscal authority (who is the sole owner of the central bank’s equity) and, therefore, we can think of \( B^m_t \) (the central bank’s holdings of government bonds) as being net of equity. In regards to the banking system, we assume that households could save in the form of bank equity or with bank deposits, but that both forms of allocating their savings to banks are perfect substitutes of each other offering the same rate of risk-free return next period \( i_t \). Hence, for simplicity, we only consider the case of
Banks funded by households entirely through bank deposits. A more complex regulatory (e.g., with capital requirements under Basel rules) or operational environment for banks, however, can make the distinction between bank capital and other sources of funding more economically relevant. We leave those issues for future research as well.

We define the Home monetary base, $MB_t$, simply as the sum of currency in circulation, $Z_t$, and the amount of required and excess reserves held by the banking system on the central bank, $U_t$. The counterpart on the asset-side of the central bank’s balance sheet are the holdings of government bonds, $B^m_t$, and the loans to commercial banks, $F_t$. What is the operational and regulatory framework under which central banks operate?

- First, the reserve requirement ratio and the return the central bank pays on reserves are some of the tools available for the conduct of monetary policy. We assume a policy framework where the return on reserves is set to zero (or at least strictly less than the risk-free rate $i_t$) in order to discourage the banking system from accumulating excess reserves on the central bank’s balance sheet. In our setting, this implies that $U_t$ is equal to the required reserves and, therefore, that more resources can be made available for bank investments since excess reserves would be zero in equilibrium. We define the required reserves as,

$$U_t = rD_t,$$

where $0 \leq r < 1$ is the reserve requirement ratio set by the policymakers. Although the reserve requirement ratio is aimed broadly to ensure that banks retain some liquidity available to safeguard their financial position while attending deposit withdrawals, in our simplified model it is interpreted more loosely as a regulatory-based constraint on the banks ability to transform deposits into credit loans for households. Under that interpretation it can also reflect other non-regulatory, but technological constraints or iceberg costs on what implicitly is a linear production function (transforming deposits directly into credit that provides liquidity gains to households).

- Second, adding or removing liquidity into the banking system, $F_t$, is another important balance sheet tool for monetary policy. We assume a monetary policy framework whereby the policymaker makes it more punishing for banks to access central bank’s loans than to fund themselves through deposits, i.e., we assume the return required on central bank loans is strictly higher than $i_t$. In this setting, banks rely entirely on deposits and use no central bank loans:

$$F_t = 0.$$  

Furthermore, this also implies that the monetary base must be equal to the central bank’s bond holdings, i.e., $MB_t = B^m_t$.

- Third, the policy framework in place also incorporates the full fiscal backing of the fiscal authority. Hence, the consolidated budget constraint of the Home country government tells us that:

$$T_t + \Delta MB_t + \int_{\omega_{t+1} \in \Omega} \left( Q_t (\omega_{t+1}) B^H_t (\omega_{t+1}) + S_t Q^*_t (\omega_{t+1}) B^{Hs}_t (\omega_{t+1}) \right) = P_t G_t + \phi W_t N_t + \left( B^H_{t-1} (\omega_t) + S_t B^{Hs}_{t-1} (\omega_t) \right),$$

where $T_t$ is the tax revenue or transfers, $\Delta B^m_t = \Delta MB_t = MB_t - MB_{t-1}$ is the seigniorage revenue from the central bank, $\int_{\omega_{t+1} \in \Omega} \left( Q_t (\omega_{t+1}) B^H_t (\omega_{t+1}) + S_t Q^*_t (\omega_{t+1}) B^{Hs}_t (\omega_{t+1}) \right)$ is the nominal amount raised from selling government state-contingent, one-period debt owned by the public in both countries (Home and
Foreign households, while \( P_t G_t \) is government spending on consumption and investments, \( \phi W_t N_t \) is the labor subsidy provided by the Home government to reverse the monopolistic competition distortion in steady state, and \( (B_{i-1}^H (\omega_t) + S_t B_{i-1}^{Ht} (\omega_t)) \) is the re-payment to the public on the contingent bonds.

We assume that the government has no expenditures apart from those that arise from subsidizing labor, i.e., we assume

\[
G_t = 0. \tag{38}
\]

We also recall that government contingent bonds are in zero net supply, i.e., market clearing implies that

\[
B_t^H (\omega_{t+1}) + S_{t+1} B_t^{Ht} (\omega_{t+1}) = 0, \forall \omega_{t+1} \in \Omega. \tag{39}
\]

In this context, the banking system will choose to invest all its deposits (except those set aside as reserves with the central bank) as long as the return on loans is higher than the risk-free rate that can be achieved with a portfolio of contingent government bonds, i.e., for any \( \omega_{t+1} \in \Omega \)

\[
B_t^H (\omega_{t+1}) = 0 \text{ if } i_{i,t} > i_t. \tag{40}
\]

We have excluded the government bond holdings from the banking system in the consolidated budget constraint of the government and the market clearing condition for government bonds precisely because in equilibrium this condition would hold (as we show later on).

We should note, however, that the Home government issues contingent bonds to the public but also non-state-contingent (and non-interest bearing) bonds which serve as the counterpart to the monetary base on the central bank’s balance sheet, i.e., \( \Delta B^m_t = \Delta M B_t \).\(^3\) This is an important part of the monetary creation process—the central bank money creation is used to fund the government, so currency (or cash) \( U_t \) is put in circulation to provide liquidity for households via fiscal policy (government transfers).

Fourth and final, as long as the bank loans achieve a rate of return \( i_{i,t} \) higher than the risk-free rate \( i_t \) that can be accrued on a portfolio of government contingent bonds, the banking system chooses to allocate all its available deposits (except required reserves) on credit loans to households. This implies simply that

\[
L_t = (1 - r) D_t. \tag{41}
\]

In this setting, the representative bank maximizes profits period-by-period since assets and liabilities have the same short maturity of one period. Under perfect competition, the banking system breaks-even (making no profits for their shareholders, the Home household) whenever it holds that

\[
i_{i,t} = \frac{1}{1 - r} i_t. \tag{42}
\]

Hence, the spread between the loan rate and the risk-free rate can be expressed as

\[
i_{i,t} - i_t = \left( \frac{r}{1 - r} \right) i_t. \tag{43}
\]

\(^3\)We could alternatively assume that these bonds \( B^m_t \) pay an interest rate of return that is rebated by the central bank in full to the fiscal authority. Hence, as long as the rebate is in full, it cancels out the corresponding payment from the fiscal authority in the consolidated budget constraint and therefore would not alter our analysis here.
which shows that the spread on loans is positive and depends on the risk-free rate (the spreads are lower when the risk-free rate is low) and the reserve requirement ratio \(0 \leq r < 1\).

**Monetary policy implementation.** In terms of monetary aggregates, it is worth noting that the monetary base, \(MB_t\), and the money supply, \(M_t\), in equilibrium are given in the model by

\[
\text{Monetary base} : \quad MB_t = Z_t + U_t,
\]
\[
\text{Money supply} : \quad M_t = Z_t + D_t,
\]

where the distinction arises from the fact that money supply includes all deposits while the monetary base only the reserves. Using the implications of the banking system balance sheet in (41), we can re-write the definition of the money supply as,

\[
M_t = Z_t + D_t = MB_t + D_t - U_t = MB_t + L_t.
\]  

(44)

In other words, the money supply is equal to a simple sum of the monetary base and the credit loans made by the banking system. Excluding bank reserves this would be a simple sum of the amounts of cash and credit available to provide liquidity services—however, as indicated before, such a simple sum is not a good measure of liquidity unless cash and credit are perfect substitutes.

From the point of view of monetary policy, monetary aggregates can be a misleading measure of liquidity in the economy. Furthermore the central bank can influence the evolution of the monetary base and the money supply by setting the currency (cash) in circulation, \(Z_t\), and the reserve requirement ratio \(0 \leq r < 1\.

We view such monetary aggregates as intermediate targets for monetary policymaking, but still retain the view that monetary policy is set not on quantities but on prices. In other words, monetary policy is set in terms of the nominal risk-free rate \(i_t\). We take the regulatory and policy framework as fixed—so, in terms of monetary policy implementation, the central bank keeps \(r\) invariant and intervenes only through the money market issuing an amount of currency (cash) \(Z_t\) sufficient to support the desired target for the short-term risk-free rate \(i_t\). We describe in more detail the Taylor (1993)-type monetary policy rule setting the target for \(i_t\) shortly.

The policy framework ensures that the spread between the loan rate and the risk-free rate is proportional to the latter (as seen in (41)). Therefore, this allows us to recover from the demand equations for real money balances and real credit balances ((9) and (11)) that,

\[
\frac{L_t}{P_t} = \frac{\mu}{1 - \mu} \left( \frac{1 - r}{r} \right)^v Z_t \frac{1}{P_t'},
\]

(45)

\[
\chi(\mu)^{1/\mu} \left( \frac{Z_t}{P_t'} \right)^{-\epsilon} = \left( \mu^{1/\mu} + (1 - \mu)^{1/\mu} \left( \frac{\mu}{1 - \mu} \right)^{1/\mu} \left( \frac{1 - r}{r} \right)^{v - 1} \right)^{\mu^{1/\mu}} (C_t)^{-\gamma} \left( \frac{i_t}{1 + i_t} \right),
\]

(46)

which shows that in equilibrium the amount of real credit that is effectively used is proportional to the real monetary balances. It also simplifies the demand for real money balances that, in this case, depends only on consumption, \(C_t\), and the risk-free rate, \(i_t\).

From here, we can go a step further relating these equilibrium conditions to conventional monetary
aggregates (which we view as intermediate targets for monetary policy). Given the definition of the money supply in (44) and the equilibrium balance sheet of the banking system in (41), it follows that:

$$M_t = Z_t + D_t = Z_t + \frac{1}{1 - r} L_t = \left[1 + \frac{1}{1 - r} \left(\frac{\mu}{1 - \mu} \left(\frac{1 - r}{r}\right)^v\right)\right] Z_t,$$

which indicates that the money supply is proportional to the currency in circulation, $Z_t$, set by the central bank. From here, we obtain that the money market and credit market equilibrium conditions can be rewritten replacing $Z_t$ with the money supply aggregate as:

$$\chi \left(\frac{(\mu)^{\frac{1}{2}} \left(1 + \frac{1}{1 - r} \left(\frac{\mu}{1 - \mu} \left(\frac{1 - r}{r}\right)^v\right)\right)^{\frac{1}{2}}}{\left(\mu^{\frac{1}{2}} + (1 - \mu)^{\frac{1}{2}} \left(\frac{\mu}{1 - \mu} \left(\frac{1 - r}{r}\right)^{v - 1}\right)\right)^{\frac{1}{2}}}\right) \left(\frac{M_t}{P_t}\right)^{-\gamma} = \frac{(C_t) - \gamma}{1 + \gamma},$$

$$L_t = \left(\frac{\mu}{1 - \mu} \left(\frac{1 - r}{r}\right)^v\right) \left(\frac{1}{1 + \frac{1}{1 - r} \left(\frac{\mu}{1 - \mu} \left(\frac{1 - r}{r}\right)^v\right)}\right) M_t,$$

These two equilibrium conditions are going to be crucial in our analysis because they pin down the behavior of credit and monetary aggregates which we observe in the data.

**Monetary policy rule.** We model monetary policy implementation via changes in $Z_t$ and set the Home country’s policy target according to a standard Wicksellian Taylor (1993)-type rule on the short-term nominal interest rate, $i_t$, i.e.,

$$1 + i_t = \frac{V_t}{\bar{V}} \left(1 + \frac{\bar{P}_t}{\bar{P}_t}\right) \left(\frac{\bar{P}_t}{\bar{P}_t}\right)^\psi_{\pi} \left(\frac{\bar{Y}_t}{\bar{Y}_t}\right)^\psi_x,$$

where $\bar{V}$ and $\bar{r} \equiv \beta^{-1}$ denote the nominal and real interest rate in steady state, $\bar{P}_t = 1$ is the deterministic steady state inflation rate, and $\psi_{\pi} > 0$ and $\psi_x \geq 0$ are the policy parameters that capture the sensitivity of the monetary policy rule to changes in inflation and the output gap, respectively. $\bar{P}_t \equiv \frac{\bar{P}_t}{\bar{P}_t} \bar{V}$ is the (gross) CPI inflation rate, $\bar{P}_t$ is the corresponding (time-varying) inflation rate target, $\bar{Y}_t$ defines the aggregate output produced in the Home country, and $\bar{Y}_t$ is the output gap in levels. Here, $\bar{V}$ defines the potential output level of the Home country and $\bar{r}_t$ is the natural (real) rate of interest—potential output and the natural rate correspond to the output and real rates that would prevail absent all nominal rigidities (but given the same realization of the shocks).

The monetary policy shock in the Home country is defined as $V_t$. Monetary shocks are described with the following bivariate stochastic process:

$$V_t = (V_t)^{1 - \delta_w} (V_{t-1})^{\delta_w} e_i^m,$$

$$V_t^* = (V_{t}^*)^{1 - \delta_w} (V_{t-1}^*)^{\delta_w} e_i^{m*},$$

$$\left(e_i^m, e_i^{m*}\right) \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_m \sigma_m \sigma_m^2 \\ \rho_m \sigma_m \sigma_m^2 & \sigma_m^2 \end{pmatrix}\right),$$
where $V$ is the unconditional mean of the process, $\delta_m$ captures the persistence, and $(\varepsilon_{m1}^n, \varepsilon_{m2}^n)^T$ is a vector of Gaussian innovations with a common variance $\sigma^2_m$ and possibly correlated across both countries $\rho_{m,m'}$.

**Optimal fiscal policy subsidy.** Monopolistic competition in production and labor introduces a distortionary steady-state price mark-up, $\theta_1$, that drives a wedge between prices and marginal costs. This steady-state distortion is a function of the elasticity of substitution across output varieties within a country $\theta \geq 1$. Home and Foreign governments raise lump-sum taxes from local households within their borders in order to subsidize labor employment and eliminate the steady-state price mark-up distortions. An optimal (time-invariant) labor subsidy proportional to the marginal cost set to be $\phi = \frac{1}{\theta}$ in every country neutralizes the steady-state monopolistic competition mark-up in the pricing rule (equation (31) in steady state).

Monetary non-neutrality arises in the model from monopolistic competition and producer currency pricing under staggered price-setting behavior à la Calvo (1983). Firms set their prices relative to a (time-varying) stochastic long-run inflation rate that is pin down in equilibrium by the central bank’s inflation target—that is, by $\bar{\Pi}_t$ in the Home country. Inflation target shocks are described with the following bivariate process:

\[
\begin{align*}
\bar{\Pi}_t &= (\bar{\Pi})^{1-\delta_\pi} (\bar{\Pi}_{t-1})^{\delta_\pi} e_{\bar{\Pi}}^t, \\
\Pi_t^* &= (\Pi)^{1-\delta_\pi} (\Pi_{t-1})^{\delta_\pi} e_{\Pi}^{t^*}, \\
\begin{pmatrix}
\varepsilon_{\Pi}^t \\
\varepsilon_{\Pi}^{t^*}
\end{pmatrix} &\sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{\Pi} & 0 \\ 0 & \sigma^2_{\Pi} \end{pmatrix} \right),
\end{align*}
\]

where $\bar{\Pi}$ is the unconditional mean of the process which we set to be zero, $\delta_\pi$ capture the persistence which is arbitrarily close to one to approximate the random walk behavior which we assume in the paper, and $(\varepsilon_{\Pi}^t, \varepsilon_{\Pi}^{t^*})$ is a vector of Gaussian innovations with a common variance $\sigma^2_{\Pi}$ and uncorrelated across both countries. Setting $\sigma^2_{\Pi}$ arbitrarily close to zero for the random walk inflation target approximates the case where the central bank targets inflation deviations relative to the model’s deterministic steady state $\bar{\Pi}$.

Although the model permits permanent shifts in the inflation target ($\bar{\Pi}_t$ and $\Pi_t^*$), the resulting inflation process remains stationary around the deterministic zero-inflation steady state (that is, at $\bar{\Pi} = 0$). Hence, this modification of the economic environment does not alter the functional form of the Phillips curve except that inflation is defined in deviations from the long-run stochastic inflation rate determined by the inflation target rather than directly in deviations from the deterministic steady state. Productivity shocks
enter into the dynamics of the model only through their impact on the dynamics of the natural (real) rates in this economy, $\tilde{\pi}_t$ and $\tilde{\pi}^*_t$. The Home and Foreign monetary shock processes $\tilde{\nu}_t$ and $\tilde{\nu}^*_t$ enter through the specification of the Taylor monetary policy rule of each country. The two countries are assumed to be symmetric in every respect, except on their consumption basket due to the assumption of home-product bias in consumption. Even so, the specification of the home-product bias is inherently symmetric as well since the share of local goods in the local consumption basket is the same in both countries and determined by the parameter $\xi$. 

## B Log-Linearized System of Equations

The open-economy New Keynesian model is summarized in Tables A1 and A2.

### Table A1 - Open-Economy New Keynesian Model: Core Equations

#### Home Economy

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phillips curve</td>
</tr>
<tr>
<td>Output gap</td>
</tr>
<tr>
<td>Monetary policy</td>
</tr>
<tr>
<td>Fisher equation</td>
</tr>
<tr>
<td>Natural interest rate</td>
</tr>
<tr>
<td>Potential output</td>
</tr>
</tbody>
</table>

#### Foreign Economy

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phillips curve</td>
</tr>
<tr>
<td>Output gap</td>
</tr>
<tr>
<td>Monetary policy</td>
</tr>
<tr>
<td>Fisher equation</td>
</tr>
<tr>
<td>Natural interest rate</td>
</tr>
<tr>
<td>Potential output</td>
</tr>
</tbody>
</table>

#### Exogenous, Country-Specific Shocks

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity shock</td>
<td>$\left( \hat{a}<em>t \right) \sim N \left( \begin{array}{c} \delta_a \ \delta</em>{a,a} \ \delta_{a,a} \ \delta_a \ \delta_{a,a} \ \delta_{a,a} \end{array} \right) \left( \begin{array}{c} \hat{a}<em>{t-1} \ \hat{a}</em>{t-1} \end{array} \right) + \left( \begin{array}{c} \sigma_a^2 \ \sigma_a^2 \ \sigma_a^2 \ \sigma_a^2 \ \sigma_a^2 \ \sigma_a^2 \end{array} \right) \left( \begin{array}{c} \hat{e}_t^a \ \hat{e}_t^a \ \hat{e}_t^a \ \hat{e}_t^a \ \hat{e}_t^a \ \hat{e}_t^a \end{array} \right) \right) $</td>
</tr>
<tr>
<td>Monetary shock</td>
<td>$\left( \hat{\gamma}<em>t \right) \sim N \left( \begin{array}{c} \delta</em>{\gamma} \ \delta_{\gamma} \ \delta_{\gamma} \ \delta_{\gamma} \ \delta_{\gamma} \ \delta_{\gamma} \end{array} \right) \left( \begin{array}{c} \hat{\gamma}<em>{t-1} \ \hat{\gamma}</em>{t-1} \end{array} \right) + \left( \begin{array}{c} \sigma_{\gamma}^2 \ \sigma_{\gamma}^2 \ \sigma_{\gamma}^2 \ \sigma_{\gamma}^2 \ \sigma_{\gamma}^2 \ \sigma_{\gamma}^2 \end{array} \right) \left( \begin{array}{c} \hat{\epsilon}_t^\gamma \ \hat{\epsilon}_t^\gamma \ \hat{\epsilon}_t^\gamma \ \hat{\epsilon}_t^\gamma \ \hat{\epsilon}_t^\gamma \ \hat{\epsilon}_t^\gamma \end{array} \right) \right) $</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Composite Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta \equiv (1 - \xi) \left[ \frac{\sigma_{\gamma}^2(\gamma(\gamma - 1)(1 - 2\xi))}{\gamma(\gamma - 1)(1 - 2\xi)} \right] $</td>
</tr>
<tr>
<td>$\Lambda \equiv 1 + (\sigma_{\gamma} - 1) \left[ \frac{\gamma(\gamma(\gamma - 1)(1 - 2\xi))}{\gamma(\gamma - 1)(1 - 2\xi)} \right] $</td>
</tr>
<tr>
<td>$\Gamma \equiv \xi \left[ \sigma_{\gamma} + (\sigma_{\gamma} - 1)(1 - 2\xi) \right] $</td>
</tr>
</tbody>
</table>
### Table A2 - Open-Economy New Keynesian Model: Non-Core Equations

<table>
<thead>
<tr>
<th>Home Economy</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td>$\tilde{y}_t = \tilde{y}_t + \tilde{x}_t$</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>$\tilde{c}_t = \Theta \tilde{y}_t + (1 - \Theta) \tilde{y}_t^c$</td>
</tr>
<tr>
<td><strong>Employment</strong></td>
<td>$\tilde{n}_t \approx \tilde{y}_t - \tilde{a}_t$</td>
</tr>
<tr>
<td><strong>Real wages</strong></td>
<td>$\tilde{w}_t - \tilde{p}_t \approx \gamma \tilde{c}_t + \phi \tilde{n}_t \approx (\varphi + \gamma \Theta) \tilde{y}_t + \gamma (1 - \Theta) \tilde{y}_t^c - \varphi \tilde{a}_t$</td>
</tr>
<tr>
<td><strong>Real Money/Credit Demand</strong></td>
<td>$\tilde{m}_t^d - \tilde{p}_t \approx \gamma \nu \tilde{c}_t - \nu \tilde{n}_t, \tilde{m}_t^d - \tilde{p}_t \approx \tilde{m}_t^d - \tilde{p}_t$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Foreign Economy</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td>$\tilde{y}_t^f = \tilde{y}_t + \tilde{x}_t^f$</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>$\tilde{c}_t^f = (1 - \Theta) \tilde{y}_t + \Theta \tilde{y}_t^c$</td>
</tr>
<tr>
<td><strong>Employment</strong></td>
<td>$\tilde{n}_t^f \approx \tilde{y}_t - \tilde{a}_t^f$</td>
</tr>
<tr>
<td><strong>Real wages</strong></td>
<td>$(\tilde{w}_t^f - \tilde{p}_t^f) \approx \gamma \tilde{c}_t^f + \phi \tilde{n}_t^f \approx \gamma (1 - \Theta) \tilde{y}_t + (\varphi + \gamma \Theta) \tilde{y}_t^c - \varphi \tilde{a}_t^f$</td>
</tr>
<tr>
<td><strong>Real Money/Credit Demand</strong></td>
<td>$\tilde{m}_t^{df} - \tilde{p}_t^f \approx \gamma \nu \tilde{c}_t - \nu \tilde{n}_t, \tilde{m}_t^{df} - \tilde{p}_t^f \approx \tilde{m}_t^{df} - \tilde{p}_t^f$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>International Relative Prices and Trade</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real exchange rate</strong></td>
<td>$\tilde{r}<em>{st} \approx (1 - 2\xi) \tilde{t}</em>{ot} t_{ot}$</td>
</tr>
<tr>
<td><strong>Terms of trade</strong></td>
<td>$\tilde{t}_{ot} \approx \left[ \frac{\gamma}{\varphi - (\gamma - 1)(1 - 2\xi)^2} \right] (\tilde{y}_t - \tilde{y}_t^f)$</td>
</tr>
<tr>
<td><strong>Home real exports</strong></td>
<td>$\tilde{e}_{xpt} \approx \Xi \tilde{y}_t + (1 - \Xi) \tilde{y}_t^f$</td>
</tr>
<tr>
<td><strong>Home real imports</strong></td>
<td>$\tilde{m}_{ipt} \approx (1 - \Xi) \tilde{y}_t - \Xi \tilde{y}_t^f$</td>
</tr>
<tr>
<td><strong>Home real trade balance</strong></td>
<td>$\tilde{t}_{bt} = \tilde{y}<em>t - \tilde{c}<em>t = (1 - \xi) \left( \tilde{e}</em>{xpt} - \tilde{m}</em>{ipt} \right) \approx (1 - \Theta) (\tilde{y}_t - \tilde{y}_t^f)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Composite Parameters</th>
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<tbody>
<tr>
<td>$\Theta \equiv (1 - \xi) \frac{\varphi - (\gamma - 1)(1 - 2\xi)}{\varphi - (\gamma - 1)(1 - 2\xi)^2}$</td>
<td></td>
</tr>
<tr>
<td>$\Xi \equiv \left( \frac{\varphi + (\gamma - 1)(1 - 2\xi)^2}{\varphi - (\gamma - 1)(1 - 2\xi)^2} \right)$</td>
<td></td>
</tr>
</tbody>
</table>

### C The Aoki-Fukuda decomposition

We use the decomposition method advocated by Aoki (1981) and Fukuda (1993) to re-express the core linear rational expectations system that characterizes workhorse model into two separate (and smaller) sub-systems for aggregates and differences. Hence, we define the world aggregate and the difference variables $\tilde{g}_t^W$ and $\tilde{g}_t^R$ as,

$$
\tilde{g}_t^W \equiv \frac{1}{2} \tilde{g}_t + \frac{1}{2} \tilde{g}_t^s, \quad (57)
$$

$$
\tilde{g}_t^R \equiv \tilde{g}_t - \tilde{g}_t^s, \quad (58)
$$

which implicitly takes into account that both countries are identical in size (with the same share of the household population and varieties located in each country). We re-write the country variables $\tilde{g}_t$ and $\tilde{g}_t^s$ as,

$$
\tilde{g}_t = \tilde{g}_t^W + \frac{1}{2} \tilde{g}_t^R, \quad (59)
$$

$$
\frac{1}{2} \tilde{g}_t = \tilde{g}_t^W - \frac{1}{2} \tilde{g}_t^R. \quad (60)
$$
If we characterize the dynamics for $\bar{g}^W_t$ and $\bar{g}^R_t$, the transformation above backs out the corresponding variables for each country $\bar{g}_t$ and $\bar{g}_t^R$.

These transformations can be applied to any of the endogenous and exogenous variables in the model. Then, under this transformation, we can orthogonalize our model into one aggregate (or world) economic system and one difference system that can be studied independently.

### C.1 Dynamics of the World Economy

It is possible to model the world economy can be described with three equations that have the same basic structure as one would find in the standard three-equation, closed-economy NK model.

The world economy NK model is described with a New Keynesian Phillips curve (NKPC), a log-linearized world Euler equation, and an interest-rate-setting rule for monetary policy. The NKPC can be cast into the following augmented form,

$$\bar{\pi}^W_t - \pi^W_t = \beta \mathbb{E}_t \left( \bar{\pi}^W_{t+1} - \pi^W_{t+1} \right) + k^W x^W_t, \quad (61)$$

where $\mathbb{E}_t(\cdot)$ refers to the expectation formed conditional on information up to time $t$, $x^W_t$ is the global output gap, $\bar{\pi}^W_t$ is global inflation, and $\pi^W_t$ is the global trend inflation. Moreover, $k^W \equiv \left( \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \right) (\varphi + \gamma) > 0$ is the slope of the global output gap that depends on the deep structural parameters of the model such as the frequency of price adjustment $0 < \alpha < 1$, and the intertemporal discount rate $0 < \beta < 1$. The NKPC describing the dynamics of aggregate world inflation arises in a two-country model with staggered price-setting à la Calvo (1983), and can be augmented to include a time-varying global trend for inflation with a standard extension to incorporate price indexation in the price-setting decision of firms as in Yun (1996). In such an environment, firms that do not re-optimize their prices would automatically increase them at the trend inflation rate of the county where they reside.

The log-linearization of the Euler equation is given by,

$$x^W_t = \mathbb{E}_t \left[ \bar{x}^W_{t+1} \right] - \frac{1}{\gamma} \left( \bar{\pi}^W_t - \mathbb{E}_t \left[ \bar{\pi}^W_{t+1} \right] - \bar{\pi}^W_t \right), \quad (62)$$

where $\bar{x}^W_t$ is the aggregate short-term nominal interest rate (an aggregate of the risk-free one-period interest rates of both countries), and $\bar{\pi}^W_t$ is the aggregate natural interest rate—the real interest rate that the economy would have experienced absent nominal rigidities, given the same realization of the real shocks. Potential output and the natural (real) interest rate are both functions of exogenous real factors (technology).

We specify a general form of the monetary policy with a Taylor (1993) rule where the central bank of each country targets their domestic short-term nominal interest rate with the same reaction function. The world Taylor rule can be cast in the following form,

$$\bar{i}^W_t = \pi_t^W + \psi \pi_t \left( \bar{\pi}^W_t - \pi_t^W \right) + \psi x^W_t + \bar{i}^W_t, \quad (63)$$

where $\pi_t^W$ is the aggregate of both countries’ central bank’s inflation target and $\bar{i}^W_t$ can be interpreted as the aggregate of each country’s central bank’s own forecast of the economy’s natural (real) interest rate. We assume that the inflation target for each country follows a random walk so that the aggregate itself, $\bar{\pi}^W_t$,
also follows a random walk, i.e.

\[ \tilde{\pi}_t^W = \tilde{\pi}_{t-1}^W + \tilde{\epsilon}_t, \]  

(64)

where \( \tilde{\epsilon}_t \) is an i.i.d. shock with zero mean. In our implementation with simulated data we collapse the target rate to a constant and normalize it to zero to be consistent with the simple scenario of a zero inflation steady state. However, we maintain the generality of the specification here for illustration purposes.

In this setting, the aggregate trend inflation \( \pi_t^W \) corresponds in equilibrium to the aggregate of the central bank’s inflation target \( \pi_t^W \). To see that, one can interpret the aggregate indexation rate \( \pi_t^W \) as the Beveridge-Nelson (stochastic) trend of the global inflation process,

\[ \pi_t^W = \lim_{j \to \infty} \mathbb{E}_t \left( \tilde{\pi}_{t+j}^W \right). \]  

(65)

The world inflation rate \( \tilde{\pi}_t^W \) in this model fluctuates around a stochastic trend given by the aggregate central bank’s inflation target. Hence, since we assume in (64) that the target is a random walk, it follows that \( \mathbb{E}_t \left( \tilde{\pi}_{t+j}^W \right) = \tilde{\pi}_t^W \) at any period \( j > 0 \). In that case, it results from the definition in (65) that \( \pi_t^W = \tilde{\pi}_t^W \) at every point in time and this confirms that trend and target inflation must be equal in equilibrium.

Using the aggregate monetary policy rule in (63) to replace \( \tilde{\pi}_t^W \) in (61) – (62), the system of equations that determines world inflation and global slack can be written in the following form,

\[ \tilde{z}_t^W = A^W \mathbb{E}_t \left( \tilde{z}_{t+1}^W \right) + a^W \left( \tilde{\pi}_t^W - \tilde{\delta}_t^W \right), \]  

(66)

where,

\[ \tilde{z}_t^W = \begin{bmatrix} \tilde{\pi}_t^W - \tilde{\pi}_t^W \\ \tilde{\delta}_t^W \end{bmatrix}, \]  

(67)

where \( A^W \) is a 2 \( \times \) 2 matrix and \( a^W \) is a 2 \( \times \) 1 matrix of structural coefficients. We assume that the process for the aggregate central bank’s predicted real rate \( \tilde{\pi}_t^W \) is stochastic and exogenous. Under the assumption that the aggregate interest rate gap \( \left( \tilde{\pi}_t^W - \tilde{\delta}_t^W \right) \) is stationary, then the system in (66) has a unique nonexplosive solution in which both \( \tilde{z}_t^W \) and \( \tilde{\pi}_t^W - \tilde{\pi}_t^W \) are stationary whenever both eigenvalues of the matrix \( A^W \) are inside the unit circle. A variant of the Taylor principle which requires that \( \psi \left( 1 - \frac{\beta}{\psi} \right) \psi > 1 \) suffices to ensure the uniqueness and existence of the nonexplosive solution for the world aggregates. Assuming this condition is satisfied, the solution can be characterized as follows,

\[ \left( \begin{array}{c} \tilde{\pi}_t^W \\ \tilde{\delta}_t^W \end{array} \right) = \left( \begin{array}{c} \tilde{\pi}_t^W \\ 0 \end{array} \right) + \sum_{j=0}^{\infty} \left( A^W \right)^j a^W \mathbb{E}_t \left( \tilde{\pi}_{t+j}^W - \tilde{\delta}_{t+j}^W \right). \]  

(68)

Hence, world inflation is determined by the world inflation target and by current and expected future discrepancies between the aggregate natural rate of interest and the aggregate of the central bank’s own target for the natural rate.

We assume that the central banks adjust their policy rule to track changes in the natural rate of interest that are forecastable one period in advance implying for the aggregate that,

\[ \tilde{\delta}_t^W = \mathbb{E}_{t-1} \left( \tilde{\pi}_t^W \right). \]  

(69)
Alternatively, we can simply assume—as most of the literature implicitly does—that $\tilde{\pi}_t^W = \tilde{r}_t^W + \tilde{\varepsilon}_t^W$, where $\tilde{r}_t^W$ corresponds to the global natural interest rate and $\tilde{\varepsilon}_t^W$ is an i.i.d. disturbance that captures non-persistent and unanticipated shocks to monetary policy. In either case, the world interest rate gap $(\tilde{r}_t^W - \tilde{\varepsilon}_t^W)$ is viewed as white noise and the solution to the global system in (66) becomes,

\[
\begin{align*}
\tilde{\pi}_t^W &= \pi_t^W + \lambda^W \left( \tilde{r}_t^W - \tilde{\varepsilon}_t^W \right) = \pi_t^W - \lambda^W \varepsilon_t^m, \\
\tilde{x}_t^W &= \mu^W \left( \tilde{r}_t^W - \tilde{\varepsilon}_t^W \right) = -\mu^W \varepsilon_t^m,
\end{align*}
\]  

(70)

(71)

where the composite coefficients $\lambda^W$ and $\mu^W$ depend on the deep structural parameters of the model. If aggregate inflation evolves as predicted by this solution, then optimal forecasts of future global inflation at any horizon $j \geq 1$ must be given by,

\[
E_t \left( \tilde{\pi}_{t+j}^W \right) = \pi_t^W = \tilde{\pi}_t^W - \frac{\lambda^W}{\mu^W} \tilde{x}_t^W,
\]

(72)

or, simply re-arranging, by,

\[
E_t \left( \tilde{\pi}_{t+j}^W - \pi_t^W \right) = -\frac{\lambda^W}{\mu^W} \tilde{x}_t^W.
\]

(73)

More generally, using the fact that $\tilde{\pi}_{t+h}^{W|t} \approx \frac{400}{h} \sum_{j=1}^h \tilde{\pi}_{t+j}^W$, we can write the forecast $h-$periods ahead as follows,

\[
E_t \left( \tilde{\pi}_{t+h}^{W|t} \right) = \frac{400}{h} \sum_{j=1}^h E_t \left( \tilde{\pi}_{t+j}^W \right) = \frac{400}{h} \sum_{j=1}^h \pi_{t+j}^W = 400 \left( \pi_t^W - \frac{\lambda^W}{\mu^W} \tilde{x}_t^W \right).
\]

(74)

This implies that no other variable should improve our forecast of changes in the global inflation if global slack and the current global inflation rate are included in the forecasting model. This feature is noted in Woodford (2008) as well, and we use it as our key identifying restriction in order to construct a reduced-form specification (an ADL model) for forecasting inflation that is consistent with the NKPC.

Forecasting future global inflation using the global output gap alone would not be accurate since global inflation potentially has a stochastic trend while global slack is stationary; one needs to include among the regressors some variable with a similar stochastic trend to that of inflation. But this need not be money growth; current global inflation itself has the same stochastic trend, so including it to forecast future inflation takes care of the trend component without the need to include any other regressors to attempt to track the stochastic trend.

What we need apart from current global inflation is additional regressors that are stationary and highly correlated with the current deviations of inflation from its stochastic trend. In theory, the global output gap is one such stationary variable with that property. More generally, what matters is which variables are most useful for tracking relatively high-frequency (or cyclical) variations in inflation. This is true regardless of the horizon over which one wishes to forecast inflation. In this sense, we find that global money can be a relevant variable to help us forecast inflation.
Proposition 1 (Proposition 2 in main text) World real money gap $\hat{m}_t^{g,W}$ is proportional to global slack,

$$\hat{m}_t^{g,W} \approx \chi \hat{\chi}_t^W,$$  

where $\chi \equiv \left(1 - \eta \left(-\psi \frac{\lambda}{\mu} + \psi \chi\right)\right)$.

Proof. The aggregate money demand equations can be expressed as follows,

$$\hat{m}_t^{d,W} - \hat{p}_t^W \approx \gamma \hat{\chi}_t^W - \nu \hat{\chi}_t^W,$$  

where aggregate world consumption is given by $\hat{c}_t^W \approx \hat{y}_t^W$. Under the solution described here and the implication that the global inflation trend and the aggregate inflation target for the central banks must equate, we know that the aggregate Taylor (1993) rule implies the following path for the nominal short-term interest rate,

$$\hat{\pi}_t^W = \pi_t^W + \left(-\psi \frac{\lambda}{\mu} + \psi \chi\right) \hat{\chi}_t^W + \hat{\nu}_t^W.$$  

We assume that $\pi_t^W + \hat{\nu}_t^W$ would be common int the actual and frictionless short-run interest rate equation and, accordingly, we can write the difference $\left(\hat{\pi}_t^W - \hat{\pi}_t^W\right)$ as $\left(\hat{\pi}_t^W - \hat{\pi}_t^W\right) = \left(-\psi \frac{\lambda}{\mu} + \psi \chi\right) \hat{\chi}_t^W$. Hence, when we express the counterpart of the aggregate money demand in (76) absent nominal rigidities, it follows that,

$$\hat{m}_t^{g,W} \equiv \left(\hat{m}_t^{d,W} - \hat{p}_t^W\right) - \left(\hat{m}_t^{d,W} - \hat{p}_t^W\right) \approx \gamma \nu \hat{\chi}_t^W - \nu \left(\hat{\pi}_t^W - \hat{\pi}_t^W\right)$$

$$\approx \gamma \nu \left(\hat{y}_t^W - \hat{y}_t^W\right) - \nu \left(\hat{\pi}_t^W - \hat{\pi}_t^W\right)$$

$$\approx \gamma \nu \left(1 - \frac{1}{\gamma} \left(-\psi \frac{\lambda}{\mu} + \psi \chi\right)\right) \hat{\chi}_t^W.$$  

C.2 Dynamics of The Difference Economy

The difference economy is described with a New Keynesian Phillips curve (NKPC), a log-linearized world Euler equation, and an interest-rate-setting rule for monetary policy. The NKPC of the difference economy

The potential economy is one where all prices are set in competitive markets and can be costlessly changed. So monetary policy only affects the nominal variables. Given that, any monetary policy would be consistent with the same allocation of resources in the frictionless economy. Therefore, to the extent that the policy framework is consistent in both the actual economy and the frictionless counterfactual, the path of the prices is the same in both cases, i.e., $\hat{\pi}_t^W = \hat{\pi}_t^W$. In this baseline scenario, the world real money gap $\hat{m}_t^{g,W}$ essentially boils down to a world nominal gap measure, $\hat{m}_t^{g,W} = \hat{m}_t^{d,W} - \hat{m}_t^{d,W}$, i.e., in our baseline we expect $\hat{m}_t^{g,W} \approx \hat{m}_t^{g,W}$ to hold. A similar argument can be extended to the global credit gap, and the real credit gap measure boils down to the nominal credit gap measure in that case too. Therefore, nominal liquidity measures are interpreted as our baseline proxies for the unobserved global slack measure.
can be cast into the following augmented form,

\[ \hat{\pi}_t^R - \bar{\pi}_t^R = \beta \mathcal{E}_t \left( \hat{\pi}_{t+1}^R - \hat{\pi}_{t+1}^R \right) + k^R \hat{\pi}_t^R, \]  

(79)

where \( \mathcal{E}_t(.) \) refers to the expectation formed conditional on information up to time \( t \), \( \hat{\pi}_t^R \) is the difference in the current output gap between the two countries, \( \hat{\pi}_t^R \) is the difference in inflation, and \( \bar{\pi}_t^R \) is the difference in trend inflation. Moreover, \( k^R \equiv \left( \frac{(1-a)(1-\beta a)}{\alpha} \right) (1-2\xi) \varphi + (2\Theta - 1) \gamma \) is the slope of the difference output gap that depends on the deep structural parameters of the model such as the frequency of price adjustment \( 0 < \alpha < 1 \), and the intertemporal discount rate \( 0 < \beta < 1 \). The NKPC describing the dynamics of the difference in inflation arises in a two-country model with staggered price-setting à la Calvo (1983) and can be augmented to include a time-varying global trend for inflation with a standard extension to incorporate price indexation in the price-setting decision of firms as in Yun (1996).

The log-linearization of the (difference) Euler equation is given by,

\[ \hat{x}_t^R = \mathcal{E}_t \left[ \hat{x}_{t+1}^R \right] - \frac{1}{\gamma} \left( \frac{(1-2\xi) + 2\Gamma}{1-2\xi} \right) \left( \hat{x}_t^R - \mathcal{E}_t \left[ \hat{x}_{t+1}^R \right] - \hat{x}_{t+1}^R \right), \]  

(80)

where \( \hat{x}_t^R \) is the difference in the short-term nominal interest rate (the difference between the risk-free one-period interest rates of each country), and \( \hat{x}_{t+1}^R \) is the difference natural interest rate.

The (difference) Taylor rule can be cast in the following form,

\[ \hat{\pi}_t^R = \hat{\pi}_t^R + \psi_\pi \left( \hat{\pi}_t^R - \hat{\pi}_t^R \right) + \psi_x \hat{x}_t^R + \hat{\varepsilon}_t^R, \]  

(81)

where \( \hat{\pi}_t^R \) is the difference between both countries’ central bank’s inflation target and \( \hat{\pi}_t^R \) can be interpreted as the difference between both country’s central bank’s own forecast of the economy’s natural (real) interest rate. We assume that the inflation target for each country follows a random walk so that the difference itself, \( \hat{\pi}_t^R \), also follows a random walk, i.e.

\[ \hat{\pi}_t^R = \hat{\pi}_{t-1}^R + \hat{\varepsilon}_t, \]  

(82)

where \( \hat{\varepsilon}_t \) is an i.i.d. shock with zero mean.

The difference trend inflation \( \hat{\pi}_t^R \) corresponds in equilibrium to the difference of the central bank’s inflation target. One can interpret the aggregate indexation rate \( \hat{\pi}_t^R \) as the Beveridge-Nelson (stochastic) trend of the differential inflation process,

\[ \hat{\pi}_t^R = \lim_{j \to \infty} \mathcal{E}_t \left( \frac{\hat{\pi}_t^R}{\hat{\pi}_{t+j}^R} \right), \]  

(83)

The differential inflation rate \( \hat{\pi}_t^R \) in this model fluctuates around a stochastic trend given by the aggregate central bank’s inflation target. Hence, since we assume in (82) that the target is a random walk, it follows that \( \mathcal{E}_t \left( \frac{\hat{\pi}_t^R}{\hat{\pi}_{t+j}^R} \right) = \hat{\pi}_t^R \) at any period \( j > 0 \). In that case, it results from the definition in (83) that \( \hat{\pi}_t^R = \hat{\pi}_t^R \) at every point in time and this confirms that trend and target inflation must be equal in equilibrium also for the differential economy.

Using the differential monetary policy rule in (81) to replace \( \hat{\pi}_t^R \) in (79) – (80), the system of equations
that determines the inflation differential and slack differential can be written in the following form,

$$\varepsilon^R_t = A^R \mathbb{E}_t \left( \varepsilon^R_{t+1} \right) + a^R \left( \bar{\varepsilon}^R_t - \bar{\varepsilon}^R_t \right),$$  

(84)

where,

$$\varepsilon^R_t \equiv \begin{bmatrix} \bar{\pi}^R_t - \bar{\pi}^R_t \\ \bar{x}^R_t \end{bmatrix},$$  

(85)

where $A^R$ is a $2 \times 2$ matrix and $a^R$ is a $2 \times 1$ matrix of structural coefficients. We assume that the process for the aggregate central bank’s predicted real rate $\bar{\nu}^R_t$ is stochastic and exogenous. Under the assumption that the interest rate gap differential $\bar{r}^R_t - \bar{\nu}^R_t$ is stationary, then the system in (66) has a unique nonexplosive solution in which both eigenvalues of the matrix $A^R$ are inside the unit circle. A variant of the Taylor principle which requires that $\psi_\pi + \left(1 - \frac{\beta}{\mu} \right) \psi_x > 1$ suffices to ensure the uniqueness and existence of the nonexplosive solution for the differential aggregates. Assuming this condition is satisfied, the solution can be characterized as follows,

$$\begin{pmatrix} \bar{\pi}^R_t \\ \bar{x}^R_t \end{pmatrix} = \begin{pmatrix} \pi^R_t \\ 0 \end{pmatrix} + \sum_{j=0}^{\infty} \left( A^R \right)^j a^R \mathbb{E}_t \left( \bar{\varepsilon}^R_{t+j} - \bar{\varepsilon}^R_{t+j} \right).$$  

(86)

Hence, the inflation differential is determined by the inflation target differential across both countries and by current and expected future discrepancies between the natural rate of interest differential and the differential of the central bank’s own target for the natural rate.

We assume that the central banks adjust their policy rule to track changes in the natural rate of interest that are forecastable one period in advance implying for the differential that,

$$\bar{\nu}^R_t = \mathbb{E}_{t-1} \left( \bar{\pi}^R_t \right).$$  

(87)

Alternatively, we can simply assume—as most of the literature implicitly does—that $\bar{\nu}^R_t = \bar{\pi}^R_t + \bar{\varepsilon}_{t+1}^u$, where $\bar{\pi}^R_t$ corresponds to the natural interest rate differential and $\bar{\varepsilon}_{t+1}^u$ is an i.i.d. disturbance that captures non-persistent and unanticipated shocks to monetary policy. In either case, the interest rate gap differential $\left( \bar{r}^R_t - \bar{\nu}^R_t \right)$ is viewed as white noise and the solution to the differential system in (84) becomes,

$$\begin{align*}
\bar{\pi}^R_t &= \pi^R_t + \lambda^R \left( \bar{\pi}^R_t - \bar{\pi}^R_t \right) = \pi^R_t - \lambda^R \bar{\varepsilon}_{t+1}^u, \\
\bar{x}^R_t &= \mu^R \left( \bar{\pi}^R_t - \bar{\nu}^R_t \right) = -\mu^R \bar{\varepsilon}_{t+1}^u,
\end{align*}$$  

(88)

(89)

where the composite coefficients $\lambda^R$ and $\mu^R$ depend on the deep structural parameters of the model. If inflation differential evolve as predicted by this solution, then optimal forecasts of future differential inflation at any horizon $j \geq 1$ must be given by,

$$\mathbb{E}_t \left( \bar{\pi}^R_{t+j} \right) = \pi^R_t = \bar{\pi}^R_t - \frac{\lambda^R}{\mu^R} \bar{x}^R_t,$$  

(90)
or, simply re-arranging, by,

$$E_t \left( \hat{\pi}^R_{t+j} - \hat{\pi}^R_t \right) = -\frac{\lambda^R}{\mu^R} \hat{x}^R_t. \quad (91)$$

More generally, using the fact that

$$\hat{\pi}^R_{t+1:t+h} \approx \frac{400}{h} \sum_{j=1}^{h} \hat{\pi}^R_{t+j},$$

we can write the forecast $h$-periods ahead as follows,

$$E_t \left( \hat{\pi}^R_{t+h|t} \right) = \frac{400}{h} \sum_{j=1}^{h} E_t \left( \hat{\pi}^R_{t+j} \right) = 400 \left( \frac{1}{h} \sum_{j=1}^{h} E_t \left( \hat{\pi}^R_{t+j} \right) - \hat{\pi}^R_t - \frac{\lambda^R}{\mu^R} \hat{x}^R_t \right). \quad (92)$$

This implies that no other variable should improve our forecast of changes in the differential inflation if differential slack and the current inflation differential rate are included in the forecasting model. This feature is noted in Woodford (2008) as well and we use it as our key identifying restriction in order to construct a reduced-form specification (an ADL model) for forecasting inflation that is consistent with the NKPC.

**D Extensions to the empirical analysis**

**D.1 The real global money and credit gap**

By Proposition 1 (Proposition 2 in main text), the global output gap is shown to be proportional to the world real money (or credit) gap in the general case where monetary policy is defined differently for the actual and frictionless economies, and therefore, the actual and potential prices of each country are determined differently as a result of the policy differences. This implies that global output gap is proportional to only global real money (or credit) gap. Following this theoretical result, we analyze if G7 average real money supply growth and G7 real credit growth helps forecast U.S. inflation.

One caveat in this empirical exercise is that PCE series is not available for all G7 countries, and therefore, we are able to deflate the nominal money (or credit) series of individual countries only by their individual CPI series. In turn, with the resulting real money and credit measures, we calculate G7 average growth rates to forecast U.S. CPI and PCE inflation. Notice that the domestic real money and credit measures (deflated by CPI) would yield the same results as the nominal measures in forecasting CPI inflation since the variables are constructed based on the first differences of the logs of these variables, and therefore we do not report the results based on domestic measures in this section. (See Figure OA1 of this Online Appendix.)

Accordingly, the G7 measure of real money growth appears to be a better predictor of CPI inflation than the credit measure. The money measure also yields more robust results with the PCE inflation forecasts than the credit measure. However, these results reinforce the validity of our theory, as global liquidity matters for inflation modeling not only in the long run but also in the short run. This result, indeed, motivates us to think of a different mechanism, i.e. an alternative interpretation of the open-economy Phillips curve, which combines the New Keynesian and Monetarist view.
D.2 Figures

Figure OA1. Evolution of the relative MSFEs of the forecasts with real global money and credit gap and the AR process of inflation. The dates on the horizontal axis indicate the end of the estimation sample for a given subsample in our forecasting experiment.
References


